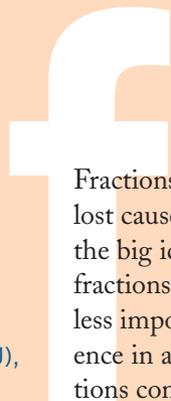




10 Practical Tips

for Making Fractions Come Alive and Make Sense

Doug M. Clarke, Anne Roche, and Annie Mitchell



Fractions are difficult to teach and to learn, but they should not be viewed as a lost cause. Important research in Australia and elsewhere can help us focus on the big ideas in ways that can help students in the middle years make sense of fractions. We share insights from our research, advice on what is important and less important, and practical classroom approaches and activities that our experience in a variety of professional development settings indicates can make fractions come alive for middle-grades teachers and students.

WHAT MAKES FRACTIONS SO DIFFICULT TO TEACH AND TO LEARN?

It is widely agreed that fractions form an important part of the middle years' mathematics curriculum (Litwiller and Bright 2002). Fractions both underpin the development of proportional reasoning and are important for future mathematics study, including that of algebra and probability. However, it is clear that many teachers find fractions difficult to understand and teach (Lamon 2007) and many students find them difficult to learn (Clarke, Roche, Mitchell, and Sukenik 2006; Pearn and Stephens 2004). Much of the confusion in teaching and learning fractions appears to arise from the many different interpretations (constructs), representations (models), and coding conventions ($5/4$, $1\ 1/4$, 1.25, 125 percent) (Kilpatrick, Swafford, and Findell 2001). Also, generalizations that have occurred



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during instruction on whole numbers have been misapplied to fractions (Streefland 1991).

Kieren (1976) identified several different interpretations (or constructs) of rational numbers often summarized as (1) part-whole, (2) measure, (3) quotient (division), (4) operator, and (5) ratio. To set the context for much of what follows, we will outline key interpretations of fractions.

The *part-whole* interpretation depends on the ability to partition either a continuous quantity (including area, length, and volume models) or a set of discrete objects into equal-sized subparts or sets.

A fraction can represent a *measure* of a quantity relative to one unit of that quantity. Lamon (1999) explained that the measure interpretation is different from the other constructs in that the number of equal parts in a unit can vary depending on how many times you partition. This successive partitioning allows you to measure with precision. We speak of these measurements as “points,” which can be modeled using a number line. A fraction (a/b) may also represent the operation of *division* or the result of a division such as $3 \div 5 = 3/5$. The division or quotient interpretation may be understood through

partitioning and equal sharing.

A fraction can be used as an *operator* to operate on a unit (or on the result of a previous operation) such as $3/4$ of $12 = 9$ and $5/4 \times 8 = 10$. The misconception that multiplication “always makes bigger” and division “always makes smaller” is common. Students’ lack of experience using fractions as operators may also contribute to this misconception.

Fractions can be used as a *ratio*—a method of comparing the sizes of two sets or two measurements, such as for every three girls there are four boys. Post, Cramer, Behr, Lesh, and Harel (1993) claimed that “ratio, measure and operator constructs are not given nearly enough emphasis in the school curriculum” (p. 328).

The dilemma for both teachers and students is how to make all the appropriate connections so that a mature and flexible understanding of fractions and the wider domain of rational numbers can be obtained. Another dilemma involves the experience of many teachers: As mathematics students themselves, they have not been equipped to teach this topic well.

Despite the difficulty of teaching and learning fractions, there are

exciting examples of teachers who are clearly supporting students who are trying to make sense of the topic. Darcy was asked by his teacher to write about or draw $3/4$ in as many ways as he could (see **fig. 1**); his work shows a breadth of understanding of a range of ways of interpreting and applying fraction knowledge.

In the remainder of this article, we offer ten tips that we think will make fractions more readily understandable for students.

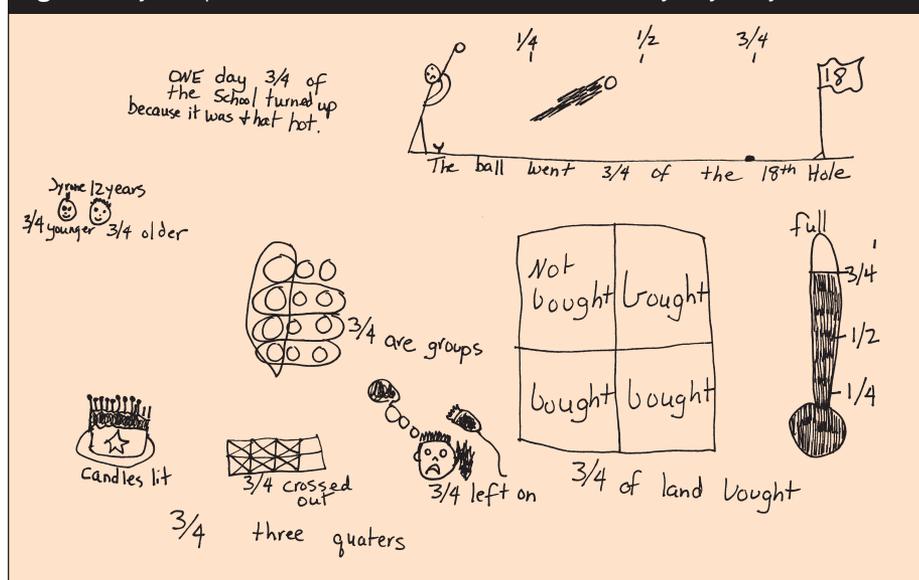
1. Give a greater emphasis to the meaning of fractions than on procedures for manipulating them.

Curriculum documents sometimes give the sense that the ultimate goal of teaching fractions is that students will be able to carry out the four operations with them. We believe that students need to be given time to understand what fractions are about (rather than moving quickly to computation) and that the ultimate goal should be to develop students who can reason proportionally.

The often-quoted result from the U.S. National Assessment of Educational Progress (NAEP) supports the claim of student difficulty in understanding the size of a fraction (Carpenter, Kepner, Corbitt, Lindquist, and Reys 1980). When estimating the answer to $12/13 + 7/8$, only 24 percent of thirteen-year-olds chose the correct answer in a multiple choice set (1, 2, 19, 21), with the majority choosing 19 or 21.

Although problems involving proportional reasoning can sometimes be solved using procedures such as $a/b = c/d$, Lamon (1999) believes that just using these equations is not reasoning and that proportional thinkers are able to (among other things) use their own vocabulary and strategies to make sense of these problems and “identify everyday contexts in which proportions are and are not useful” (p. 235).

Fig. 1 Darcy’s response to “write or draw about $3/4$ in as many ways as you can.”



We want students who can look at an advertisement for gasoline discounts in Australia claiming that a “5 percent discount beats four cents a litre discount” and decide when this statement is and is not true. We want students to understand that the ratio of surface area to volume as well as dehydration are the mathematical reasons why a baby left in a car on a hot day will suffer when an adult in the same circumstances would not (Lovitt and Clarke 1988). We also want students to use number sense and proportional reasoning to identify the best value from the two detergent choices in **figure 2**.

2. Develop a generalizable rule for explaining the numerator and denominator of a fraction.

When students are first trying to make sense of common fractions, teachers have typically defined a fraction as follows:

The denominator tells you how many parts the whole has been broken up into, and the numerator tells you how many of these parts to take, count, or shade in.

This explanation works reasonably well for fractions between 0 and 1 but not for improper fractions, which are fractions greater than 1. We prefer *this* explanation for students: In the fraction a/b , b is the name or size of the part (e.g., fifths have this name because 5 equal parts can fill a whole), and a is the number of parts of that name or size. If we have $7/3$, the 3 tells the name or size of the parts (thirds) and the 7 tells us that we have 7 of those thirds (or $2\frac{1}{3}$).

We believe that this new rule may help students use more appropriate language when labeling fractions. For example, we have noticed that some students refer to three-quarters as “three-fours” and “four-threes.” This



use of whole-number rather than fractional language appeared to be an indicator that the students do not yet understand which digit refers to the number of parts or the size of the parts.

3. Emphasize that fractions are numbers, making extensive use of number lines in representing fractions and decimals.

Kilpatrick, Swafford, and Findell (2001) commented that the fact that rational numbers are numbers is so fundamental that it is easily overlooked.

Using a number line has many advantages. It helps students see how whole numbers, fractions, and decimals relate; it provides a way of understanding why $5/3$ is the same as $1\frac{2}{3}$ and that $6/3$ is the same as 2, and it makes it easier for students to understand the notion of the density of rational numbers (i.e., that between any two distinct fractions or decimals, there is an infinite number of fractions and decimals).

4. Take opportunities early to focus on improper fractions and equivalences.

If students seem to understand the language of fractions and are making good use of number lines, then improper fractions will fit in quite naturally. Color in Fractions (see the **activity sheet**) develops both an understanding of equivalence and of the meaning of improper fractions. Take, for example,

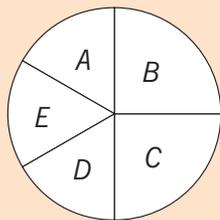
students’ making sense of $4/3$ in the context of this game. The challenge of being the first to completely color their game board motivates them to think about what is equivalent to $4/3$, such as a 1 row plus $1/3$, or $5/6$ and $1/2$. This game is accessible to everyone. Students who are struggling with equivalence have to color just what they roll (e.g., “I rolled $2/6$, so I’ll color in 2 of the sixths”), but they soon observe their partners using equivalence and pick up on the idea and the potential flexibility, particularly if they think they are going to miss a turn because they cannot find equivalent areas to color.

5. Provide a variety of models to represent fractions.

A range of manipulatives and other tools have been employed during teaching experiments (Post, Wachsmuth, Lesh, and Behr 1985; Steencken and Maher 2002), such as fraction bars, Cuisenaire rods, paper folding, laminated shapes, and computer programs. Ball (1993), however, advised that ready-made fraction materials may not enable students to construct important concepts regarding fractions, such as that the unit must be the same size when comparing or that equal shares are not necessarily congruent. Empson (2002) found that posing problems involving equal-sharing tasks provided opportunities for students to generate their own models, which embodied many crucial aspects of fractions.

Overall, if students are to become flexible in moving between different constructs, they need to be familiar with different representations (and manipulatives), as each model differs in its ability to reflect each construct or concept under investigation. Sowder (1988) noted that when it comes to fractions, students are “model poor,” with many viewing a circular region as the only model of a fraction. This was our experience as well, when considering students’

Fig. 3 Pie task



- a. What fraction of the circle is part B?
- b. What fraction of the circle is part D?

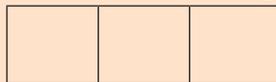
responses to “write or draw about $3/4$ in as many ways as possible” discussed earlier in the article.

With the one-on-one interview we developed for fractions and decimals (see Clarke et al. 2006), we used a task from the Rational Number Project about interpreting a circular model. Students were presented with the task shown in **figure 3** (adapted from Cramer, Behr, Post, and Lesh 1997).

Part (a) was relatively straightforward, with 83.0 percent of grade 6 students answering $1/4$. A further 3.4 percent offered a correct equivalent fraction, decimal, or percent, whereas 5.6 percent and 1.9 percent answered $1/5$ and $1/2$, respectively ($n = 323$). Part (b) was more difficult, with only 42.7 percent giving the correct answer of $1/6$ and 13.6 percent answering $1/5$ (presumably based on “five parts”). The same percent of students answered $1/3$, possibly focusing only on the left-hand side. These students probably had limited experience with partitioning involving nonequal parts and may have had little exposure to area-model tasks containing “perceptual distractors” (Behr and Post 1981). We believe that students will benefit from tasks that have visually distracting elements like that shown in **figure 4**. The students will then be forced to apply mathematical thinking rather than simply counting and shading. This type of task has also been used

Fig. 4 A task containing a “perceptual distractor”

Shade $3/4$ of the rectangle



to assess student understanding of decimals (Roche 2005).

6. Link fractions to key benchmarks, and encourage estimation.

In our one-on-one interview, we provided students with eight pairs of fractions and asked them to decide, for each pair, which fraction was larger and why (see **fig. 5**). No paper was provided—the students were required to do the comparison in their heads. Try this task for yourselves. Can you choose the larger of each pair, and how did you decide? Please record your responses before reading on.

The tasks were given to 323 grade 6 students at the end of the school year. Performance on the pairs varied, with 77.1 percent choosing correctly (with appropriate explanation) for the pair ($3/8, 7/8$), down to the hardest pair ($3/4, 7/9$), for which only 10.8 percent were successful.

Although common denominators were used (with mixed success) by a number of students on many pairs, the most successful students used

two strategies that they may *not* have been taught in school. The first we call *benchmarking*: Students compare the size of fractions with 0, $1/2$, or 1. For the pair ($3/7, 5/8$), students might note that $3/7$ is less than one-half and that $5/8$ is more than one-half. It was clear that these students had tip 3 under control.

Another innovative strategy was what we call *residual* thinking: Students refer to the amount required to build up to the whole. In comparing $5/6$ and $7/8$, students may conclude that the first fraction requires $1/6$ more to make the whole (“the *residual*”), whereas the second requires only $1/8$ to make the whole (a smaller piece or number), so $7/8$ is larger. This is excellent student reasoning.

On the other hand, some students use an inappropriate strategy by considering the difference, in whole numbers, between the numerator and denominator (see *gap thinking* as described in Pearn and Stephens 2004) and argue that $5/6$ and $7/8$ are equivalent, since they both require one “bit” to make a whole.

We believe that when students share their strategies for comparing fractions during class, other students may be convinced to use both benchmarking and residual thinking to tackle relative-size and ordering problems. It is not that common

Fig. 5 A fraction pairs task

Fraction Pairs	Which Fraction Is Larger? Record Your Strategy
a. $3/8$ $7/8$	
b. $1/2$ $5/8$	
c. $4/7$ $4/5$	
d. $2/4$ $4/8$	
e. $2/4$ $4/2$	
f. $3/7$ $5/8$	
g. $5/6$ $7/8$	
h. $3/4$ $7/9$	

denominators do not work, it is that benchmarking and residual strategies are often more efficient and focus on *meaning* in their application, something so often missing from the school fraction experience of students.

Surveys completed by two hundred adults over a twenty-four-hour period found that more than 60 percent of all calculations carried out in daily life only required an estimate (Northcote and McIntosh 1999). We believe that the curriculum emphasis should reflect this finding. This is one reason why teaching fraction algorithms for the four operations does not prepare students for real-life encounters with fractions, where mental estimation is the key skill.

7. Give emphasis to fractions as division.

The notion of “fraction as division” is not a common construct in most people’s minds (see Clarke 2006). If we understand, for example, that one meaning of $2/3$ is “2 divided by 3,” then strategies in sharing situations become obvious quite quickly. Ironically, in teaching students to convert $3/7$ to a decimal, we encourage them to use a calculator to divide the 3 by the 7, thus invoking the construct of fractions as division, without explaining why. Similarly, $17/5$ can be represented as the mixed number $3 \frac{2}{5}$, and students tend to convert it by dividing 17 by 5—the same principle again.

In our interview, students were shown a picture similar to that from Lamon (2007) of five girls and three pizzas, and were asked, “Three pizzas were shared equally among 5 girls. How much pizza does each girl get?” Although 30.3 percent of grade 6 students responded with a correct answer ($3/5$), it was apparent that most either drew a picture or mentally divided the pizzas to calculate the equal share, indicating that “3 shared among 5 is $3/5$ ” is not an automatic understanding and needed to be worked out by

partitioning. Another result of concern was that 11.8 percent of students were unable to get started. Students need greater exposure to division problems and explicit discussion connecting division with their fraction answers (e.g., $3 \div 5 = 3/5$ may lead to the generalization that $a \div b = a/b$).

8. Link fractions, decimals, and percents wherever possible.

Many middle school students, when given a problem to solve involving fractions, will choose to convert it to decimals or percents to make sense of it. This flexible thinking is to be encouraged, as percentages particularly seem to make sense to many students intuitively. A number of researchers believe that decimals and percentages should be introduced far earlier than many teachers typically do (see, e.g., Moss and Case 1999).

The following activity helps students make connections among fractions, decimals, and percents. The set of cards shown in **figure 6** was adapted from Eggleton and Moldavan (2001). Give each student a card and ask him or her to line up in front of the classroom, arranging themselves in order from smallest to largest. The set of cards is quite difficult for many students to arrange in order, but teachers can of course adapt the set to the needs of the class. As noted earlier, being able to translate among different representations may enable the student to get a clearer understanding of the different constructs of rational numbers.

9. Take the opportunity to interview several students one on one on the kinds of tasks discussed in this article to gain awareness of their thinking and strategies.

We have discussed several tasks that we have used in a one-on-one interview setting as part of our research. We have also encouraged teachers to try a number of these activities with

Fig. 6 A set of cards connecting fractions, decimals, and percents

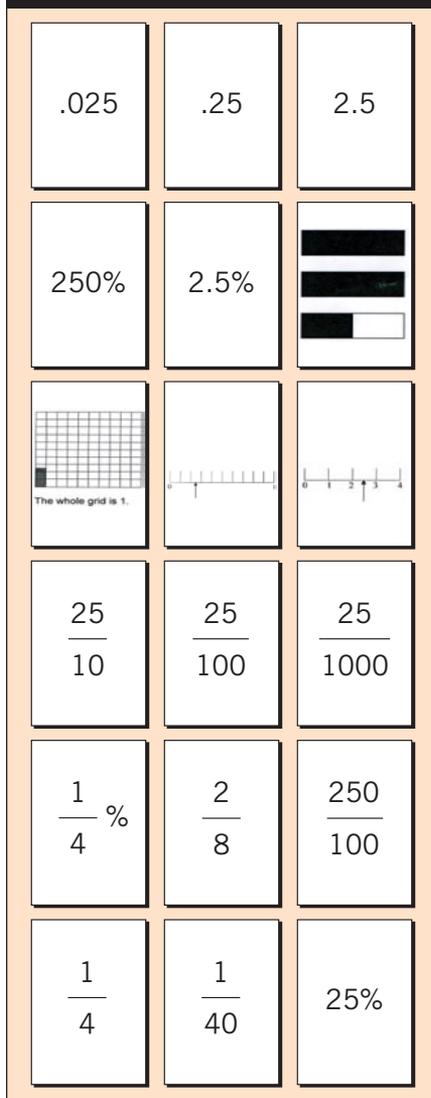
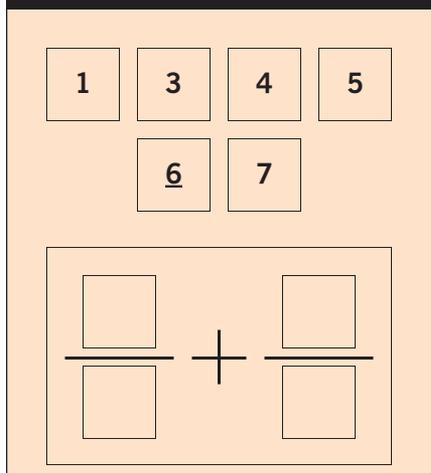


Fig. 7 The cards and board for the Construct a Sum activity



their students. In almost every case, teachers report that the interviews were particularly useful in gaining insights into their students' thinking. In many cases, the interview process developed a new respect for students' attempts to make sense of fractions. They were eager to share students' methods with their other students and to encourage them to try these strategies with a range of other problems.

One of our favorite tasks, whether in interview format or as a whole-class activity, is "Construct a Sum" from the Rational Number Project (Behr, Wachsmuth, and Post 1984). Students are given cards containing the numbers 1, 3, 4, 5, 6, and 7 and a board (see **fig. 7**) and asked to place the number cards inside the boxes so that the total is close to but not equal to 1. Only 25.4 percent of our grade 6 interviewees created a combination in the range 0.9 to 1.1, with a total of 85 different combinations offered. In addition, 24.4 percent of students created at least one improper fraction in their solution (making the result more than 1 with still more to add), perhaps indicating a misunderstanding about the size of improper fractions. This task provides a wonderful basis for discussing relative size and an opportunity to address students' understanding of improper fractions.

10. Look for examples and activities that can engage students in thinking about fractions in particular and rational number ideas in general.

We would like to share briefly some of our favorite fraction activities.

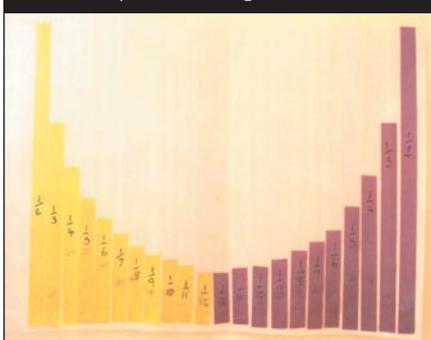
Fraction suspension bridge (adapted from the New South Wales Department of Education and Training 2003). After showing pictures of various kinds of bridges, show students a photograph of the Golden Gate Bridge in San Francisco and explain that they are going to construct something similar

using fractions. Give each pair of students three 60 cm strips of ribbon or paper and ask them to consider how the ribbon could be cut to represent these fractions— $1/2$, $1/3$, $1/4$, $1/5$, $1/6$, $1/7$, $1/8$, $1/9$, $1/10$, $1/11$, and $1/12$ —given that each strip represents 1 whole unit. Each pair makes these fractions from three strips, then combines their fractions with another pair of students to create a suspension-like bridge (see **fig. 8**). It is not necessary to use all pieces of the ribbon. This activity provides good practice with problem solving as well as practice using the fraction as an operator to find, for example, $1/6$ of 60 cm. Students are also quite interested, as we were, in the visual proof of how the difference between successive unit fractions decreases as we move from $1/2$ to $1/3$, to $1/4$, and so on.

Cuisenaire rods (moving from part to whole, whole to part, and part to part). Each group of students has a box of Cuisenaire rods that they use to solve problems like the following:

- What fraction of the brown rod is the red rod?
- If the purple is $2/3$, which rod is the whole?
- If the brown rod is $4/3$, which rod is 1?
- If the blue rod is $1\ 1/2$, which rod is $2/3$?
- What question do you want to ask the group?

Fig. 8 Students use their ribbons to produce a suspension bridge out of fractions.



Sticky numbers. Write a fraction, decimal, or percent on a sticky note and place it on the back of each student. Students then move around the room, asking one person at a time a question until they discover their mystery number. Each question can only be answered yes or no. Desirable questions include benchmarking ("Am I less than a half?"), since questions that isolate the numerator and denominator ("Is my top number odd?") are unlikely to promote the kind of thinking that we are advocating.

Making sense of operations. Students are encouraged to draw a picture or use materials to give some meaning to each of the following expressions: $1/2 + 1/3$, $1/2 - 1/3$, $1/2 \times 1/3$, and $1/2 \div 1/3$.

CONCLUSION

In this article, we have outlined the many challenges facing teachers and students in schools and colleges in the teaching and learning of important fraction concepts. A range of big ideas has been discussed; some important data from one-on-one interviews with students have been shared; and ten practical, research-based tips for the classroom teacher have been offered, with appropriate classroom activities suggested. We are confident from our experience in professional development settings that these ideas and suggestions have great potential for making fractions come alive and make sense in the middle years.

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